

Magnetic hard-axis ordering near ferromagnetic quantum criticality

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We investigate the interplay of quantum fluctuations and magnetic anisotropies in metallic ferromagnets. Our central result is that fluctuations close to a quantum critical point can drive the moments to point along a magnetic hard axis. As a proof of concept, we show this behavior explicitly for a generic two-band model with local Coulomb and Hund's interactions, and a spin-orbit-induced easy plane anisotropy. The phase diagram is calculated within the fermionic quantum order-by-disorder approach, which is based on a self-consistent free energy expansion around a magnetically ordered state with unspecified orientation. Quantum fluctuations render the transition of the easy-plane ferromagnet first-order below a tricritical point. At even lower temperatures, directionally dependent transverse fluctuations dominate the magnetic anisotropy and the moments flip to lie along the magnetic hard axis. We discuss our findings in the context of recent experiments that show this unusual ordering along the magnetic hard direction.

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Fluctuations near to quantum critical points in itinerant ferromagnets (FMs) can have drastic and often surprising effects. They generically render *a priori* continuous phase transitions first order at low temperatures [1–5]. In many systems, quantum phase transitions are preempted by the formation of superconducting [5–7], modulated magnetic [8], or unusual spin-glass phases [9, 10]. Other metallic FMs show an unexpected ordering along the magnetic hard axis [11, 12].

It is well understood that the first-order behavior arises from the coupling of the magnetic order parameter to soft electronic particle-hole fluctuations, giving rise to non-analytic terms in the free energy [13–16]. Because of this interplay between low-energy quantum fluctuations, metallic FMs are very susceptible towards the formation of incommensurate magnetic [17], spin nematic [14] or modulated superconducting states [18]. This spatial modulation is associated with deformations of the Fermi surface that enhance the phase space for low energy particle-hole fluctuations. The phase reconstruction can therefore be viewed as a *fermionic* quantum order-by-disorder effect, which can be studied systematically by self-consistently calculating fluctuations around a whole class of possible broken-symmetry states [19–21].

The coupling to electronic quantum fluctuations can also have counter-intuitive effects upon the direction of the magnetic order parameter. A notable example is the partially ordered phase of the helimagnet MnSi, in which the spiral ordering vector rotates away from the lattice favored directions [22, 23]. Similar effects are possible in homogenous itinerant FMs. This is suggested by recent experiments that show unusual ordering of magnetic moments along hard magnetic directions [11, 12].

The first example is YbRh₂Si₂, which is a prototypical system for studying antiferromagnetic quantum crit-

icality, but exhibits strong FM fluctuations [24]. Interestingly, isoelectronic cobalt substitution for rhodium or hydrostatic pressure stabilize FM order along the hard axis at low temperatures [11, 25]. This behavior has been interpreted as a combined effect of magnetic frustration and classical fluctuations [26]. Quantum fluctuations close to a quantum critical point are potentially strong enough to drive a moment reorientation even in the absence of frustration. This is supported by recent experiments on YbNi₄P₂ [12], which show quantum critical fluctuations with a uniform magnetic susceptibility that is an order of magnitude larger along the easy direction. Remarkably, at very low temperatures – well below the Kondo temperature – this system displays a switch of magnetic response anisotropy similar to YbRh₂Si₂ and develops FM order along the hard direction.

In this Letter, we demonstrate that fluctuation-driven moment reorientation from the easy towards the hard magnetic direction is a very generic phenomenon that is expected to occur in a large variety of itinerant FMs close to quantum criticality. The underlying mechanism has the same origin as the fluctuation-driven first-order behavior seen in practically all itinerant FMs at low temperatures, irrespective of microscopic details [16]. At mean-field level, spin-orbit (SO) coupling leads to magnetic anisotropy $F_{\text{ani}} \sim (m_{\parallel}^2 - m_{\perp}^2)$ with m_{\parallel} and m_{\perp} the moment components along hard and easy directions, respectively. The coupling of the magnetic order parameter to soft electronic particle-hole fluctuations gives rise to a directional dependent, non-analytic free-energy contribution $\delta F_{\text{ani}} \sim m^2(m_{\parallel}^2 - m_{\perp}^2) \ln(T/\mu)$, that competes with the mean-field anisotropy and dominates at sufficiently low temperatures. The resulting switching of moments toward the magnetic hard axis opens up phase space for low energy, transverse spin fluctuations.

Our starting point is a generic two-band model with local intra-band Hubbard repulsion U , Hund's coupling J_H , and SO coupling λ between the bands,

$$H = \sum_{\alpha=1,2} \sum_{\nu=\uparrow,\downarrow} \int_{\mathbf{k}} \epsilon_{\alpha}(\mathbf{k}) c_{\mathbf{k}\alpha\nu}^{\dagger} c_{\mathbf{k}\alpha\nu} + U \sum_{\mathbf{r},\alpha} \hat{n}_{\mathbf{r}\alpha\uparrow} \hat{n}_{\mathbf{r}\alpha\downarrow} - J_H \sum_{\mathbf{r}} \hat{\mathbf{S}}_{\mathbf{r}1} \cdot \hat{\mathbf{S}}_{\mathbf{r}2} + \frac{\lambda}{2} \int_{\mathbf{k}} \left(c_{\mathbf{k},1\uparrow}^{\dagger} c_{\mathbf{k},2\downarrow} - c_{\mathbf{k},1\downarrow}^{\dagger} c_{\mathbf{k},2\uparrow} + \text{h.c.} \right), \quad (1)$$

where the operators $c_{\mathbf{k}\alpha\nu}^{\dagger}$ ($c_{\mathbf{k}\alpha\nu}$) create (annihilate) an electron with momentum \mathbf{k} and spin ν in band α , and $\hat{n}_{\mathbf{r}\alpha\nu} = c_{\mathbf{k}\alpha\nu}^{\dagger} c_{\mathbf{k}\alpha\nu}$ and $\hat{\mathbf{S}}_{\mathbf{r}\alpha} = \frac{1}{2} \sum_{\nu\nu'} c_{\mathbf{r}\alpha\nu}^{\dagger} \boldsymbol{\sigma}_{\nu\nu'} c_{\mathbf{r}\alpha\nu'}$ denote the occupation-number and electron-spin operators, respectively. Here $\boldsymbol{\sigma}_{\nu\nu'} = (\sigma_x, \sigma_y, \sigma_z)_{\nu\nu'}$, with σ_i the standard Pauli matrices. For simplicity, we assume isotropic electron dispersions $\epsilon_1(\mathbf{k}) = tk^2$ and $\epsilon_2(\mathbf{k}) = \epsilon_1(\mathbf{k}) + \Delta$ with a crystal field splitting Δ . Tight-binding corrections to the dispersion do not lead to magnetic anisotropies or qualitatively change the phase diagram, as long as the system is far from instabilities due to nesting or van-Hove singularities [19]. The above SO term is of the standard form for an orbital multiplet that transforms as an $L = 1$ angular momentum (e.g. the three t_{2g} orbitals) [27], projected onto the two low-lying bands [28].

We first explain the *mean-field behavior of the free energy*. For sufficiently large U , the ground state of the system is a FM, and the directions of the magnetizations \mathbf{m}_1 and \mathbf{m}_2 of the two bands are locked together by the Hund coupling J_H . Without SO interaction ($\lambda = 0$), the mean-field free energy is independent of the direction of the total magnetization,

$$F_{\text{mf}}^{(0)} = \frac{1}{1 - \left(\frac{J_H}{2U}\right)^2} [U(\tilde{m}_1^2 + \tilde{m}_2^2) - J_H \tilde{m}_1 \tilde{m}_2] + \sum_{\alpha} (A_{\alpha} \tilde{m}_{\alpha}^2 + B_{\alpha} \tilde{m}_{\alpha}^4 + C_{\alpha} \tilde{m}_{\alpha}^6). \quad (2)$$

Here we have introduced the weighted moments $\tilde{m}_{\alpha} = m_{\alpha} + \frac{J_H}{2U} m_{\bar{\alpha}}$, with $\bar{\alpha} = 2$ for $\alpha = 1$ and vice versa. The coefficients in this Landau expansion are given by $A_{\alpha} = U^2 \int_{\mathbf{k}} n_F'[\epsilon_{\alpha}(\mathbf{k})]$, $B_{\alpha} = \frac{U^4}{12} \int_{\mathbf{k}} n_F'''[\epsilon_{\alpha}(\mathbf{k})]$, and $C_{\alpha} = \frac{U^6}{360} \int_{\mathbf{k}} n_F^{(5)}[\epsilon_{\alpha}(\mathbf{k})]$ with n_F the Fermi function. Due to the crystal field splitting, Δ , between the two bands, we find two distinct magnetic transitions that are separated in temperature. Since we are interested in behavior near to the paramagnetic state, for large enough Δ we approximate the free energy, keeping only the classical contributions to the $\alpha = 2$ band. In this case, $\tilde{m}_2 = \frac{J_H}{2U} \tilde{m}_1$.

The directional dependence arises from SO coupling, which we treat in second-order perturbation theory, assuming $\lambda/2\Delta \ll 1$. This gives us an additional term

$$F_{\text{mf}}^{(\lambda)} = \frac{\lambda^2}{2U^2} B_1 (\mathcal{P}_{\parallel} - \mathcal{P}_{\perp}) \tilde{m}_1 \tilde{m}_2, \quad (3)$$

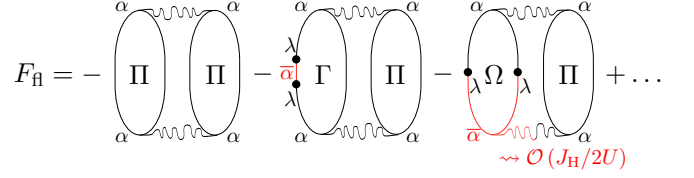


FIG. 1: (color online) Feynman diagrams for the fluctuation contributions to the free energy up to quadratic order in the SO coupling λ . Solid lines indicate fermionic Green functions in the presence of FM order, wiggly lines denote propagators of the fluctuation fields.

where we have defined the projectors onto the y -axis $\mathcal{P}_{\parallel} = n_y^2$ and the xz -plane $\mathcal{P}_{\perp} = n_x^2 + n_z^2 = 1 - n_y^2$ in terms of a unit vector \mathbf{n} which parametrizes the magnetization direction $\mathbf{m}_{\alpha} = m_{\alpha} \mathbf{n}$. Since $B_1 > 0$ we obtain an easy-plane anisotropy at mean-field level.

To derive an expression for the *fluctuation contribution to the magnetic free energy*, we first express the action as a fermion coherent-state path integral over Grassmann fields $\Psi(\mathbf{r}, \tau) = [\psi_{\uparrow}(\mathbf{r}, \tau), \psi_{\downarrow}(\mathbf{r}, \tau)]$. To decouple the interaction terms, we perform a Hubbard-Stratonovich transformation, introducing spin- and charge-fluctuation fields $\phi_{\alpha}(\mathbf{r}, \tau)$ and $\rho_{\alpha}(\mathbf{r}, \tau)$. This leads to the action

$$\mathcal{S} = \int_0^{\beta} d\tau \int d^3\mathbf{r} \left\{ U \sum_{\alpha} (\phi_{\alpha}^2 - \rho_{\alpha}^2) + J_H \phi_1 \cdot \phi_2 + \sum_{\alpha} \bar{\Psi}_{\alpha} [\partial_{\tau} - t \nabla^2 - \mu_{\alpha} + U(\rho_{\alpha} - \boldsymbol{\varphi}_{\alpha} \cdot \boldsymbol{\sigma})] \Psi_{\alpha} + i \frac{\lambda}{2} (\bar{\Psi}_1 \sigma_y \Psi_2 - \bar{\Psi}_2 \sigma_y \Psi_1) \right\}, \quad (4)$$

where we have defined $\mu_1 = \mu$, $\mu_2 = \mu - \Delta$, and $\boldsymbol{\varphi}_{\alpha} := \boldsymbol{\phi}_{\alpha} + \frac{J_H}{2U} \boldsymbol{\phi}_{\bar{\alpha}}$. The magnetizations of the bands are given by the zero-frequency components of the spin-fluctuation fields, $\mathbf{m}_{\alpha} = \boldsymbol{\phi}_{\alpha}(\mathbf{r}, \omega = 0)$.

The key idea behind the fermionic quantum order-by-disorder approach is to include the magnetic order parameter in the free-fermion propagator and to self-consistently expand around the magnetically ordered, broken symmetry state. In this particular case, we expand around a magnetic state with an arbitrary magnetization direction, parametrized by the unit vector $\mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. The resulting free-fermion action is easily diagonalized by a rotation to new fermion fields $\eta_{\alpha} = \exp(i\theta \sigma_y/2) \exp(i\phi \sigma_z/2) \Psi_{\alpha}$, leading to $\mathcal{S}_0 = \sum_{\alpha\nu} \sum_{\mathbf{k}} G_{\alpha,\nu}^{-1} \bar{\eta}_{\alpha,\nu} \eta_{\alpha,\nu}$ with Green's function

$$G_{\alpha\nu}(\mathbf{k}, \omega) = [i\omega - (\epsilon_{\alpha}(\mathbf{k}) - \nu U \tilde{m}_{\alpha} - \mu)]^{-1}. \quad (5)$$

In this new basis, the SO terms explicitly depend upon the moment direction \mathbf{n} . We perform the Gaussian integral over the fermion fields, keeping terms up to quadratic order in the finite-frequency fluctuation fields. We then

integrate over the fluctuation fields to obtain

$$F_{\text{fl}} = -\frac{1}{2}U^2T \sum_{\alpha, \bar{\omega}} \sum_{\nu_1, \nu_2} \int_{\mathbf{q}} \left\{ \Pi_{\nu_1, \nu_2}^{(\alpha)} \Pi_{\bar{\nu}_1, \bar{\nu}_2}^{(\alpha)} + \lambda^2 \Pi_{\nu_1, \nu_2}^{(\alpha)} \left(\mathcal{P}_{\parallel} \Gamma_{\bar{\nu}_1, \bar{\nu}_2, \bar{\nu}_2}^{(\alpha)} + \mathcal{P}_{\perp} \Gamma_{\bar{\nu}_1, \bar{\nu}_2, \nu_2}^{(\alpha)} \right) \right\} + \dots, \quad (6)$$

where, as before, we keep only terms up to quadratic order in λ . The corresponding Feynman diagrams are shown schematically in Fig. 1. Since the third diagram couples fluctuation fields from different bands it only contributes at higher-order in $J_H/2U$. We therefore neglect this term in the calculations below. In terms of the fermionic Green functions (5), the fermionic bubble diagrams are given by

$$\Pi_{\nu_1 \nu_2}^{(\alpha)}(q) = T \sum_{\omega} \int_{\mathbf{k}} G_{\alpha \nu_1}(k+q) G_{\alpha \nu_2}(k) \quad (7)$$

$$\Gamma_{\nu_1 \nu_2 \nu_3}^{(\alpha)}(q) = T \sum_{\omega} \int_{\mathbf{k}} G_{\alpha \nu_1}(k+q) G_{\alpha \nu_2}^2(k) G_{\bar{\alpha}, \nu_3}(k) \quad (8)$$

where we have defined $k = (\mathbf{k}, \omega)$ for brevity.

For $\lambda = 0$, the bands decouple and F_{fl} (6) reduces to the expression for the single band case [17, 19]. After summation over Matsubara frequencies and re-summation of the leading divergencies in temperature to all orders in the magnetization we obtain [21]

$$F_{\text{fl}}^{(0)} = \frac{c_1}{2} \frac{U^6}{\mu^5} \tilde{m}_1^4 \ln \left(\frac{\kappa U^2 \tilde{m}_1^2 + T^2}{\mu^2} \right) - c_2 \frac{U^4}{\mu^3} \tilde{m}_1^2, \quad (9)$$

in agreement with the result of Belitz and Kirkpatrick [13]. We have neglected the fluctuation corrections for the second band, which is far from a magnetic instability, and defined $c_1 = \frac{16}{3} \sqrt{2} (2\pi)^{-6} (\mu/t)^{9/2}$, $c_2 = (1 + \ln 2) c_1$. Since fluctuations give rise to a $\ln T$ -contribution to the m^4 coefficient, the transitions turn first-order below a certain tri-critical temperature T_c .

Although the fluctuation terms of order λ^2 appear to be much harder to calculate we can use a trick to rewrite $\Gamma_{\nu_1 \nu_2 \nu_3}^{(\alpha)}(q)$ in terms of a generalized derivative of $\Pi_{\nu_1 \nu_2}^{(\alpha)}(q)$ [29]. From this, it is straightforward to see that the dominant directional-dependent fluctuation terms originate from *transversal* spin fluctuations and are proportional to a derivative of $F_{\text{fl}}^{(0)}$,

$$F_{\text{fl}}^{(\lambda)} = \frac{\lambda^2}{8\Delta^2} (\mathcal{P}_{\parallel} - \mathcal{P}_{\perp}) \tilde{m}_2 \partial_{\tilde{m}_1} F_{\text{fl}}^{(0)}. \quad (10)$$

Since the magnetization of the system (global minimum of $F_{\text{mf}} + F_{\text{fl}}$) is always smaller than the value at which the function $F_{\text{fl}}^{(0)}$ has its minimum, the derivative in Eq. (10) is negative. Hence the directionally-dependent fluctuation terms compete with the mean-field anisotropy and could potentially stabilize FM order along the hard axis at sufficiently low temperatures.

We calculate the *phase diagram* by minimizing the free energy $F = F_{\text{mf}}^{(0)} + F_{\text{mf}}^{(\lambda)} + F_{\text{fl}}^{(0)} + F_{\text{fl}}^{(\lambda)}$. For small SO

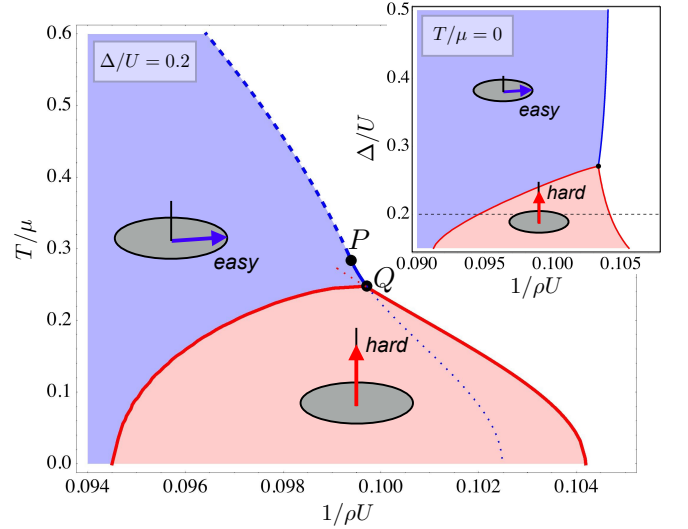


FIG. 2: (color online). Phase diagram as a function of temperature T/μ and inverse interaction strength $1/\rho U$ (ρ is proportional to the density of states of band $\alpha = 1$ at the Fermi level) for $J_H/2U = 0.3$, $\lambda/U = 0.2$, and $\Delta/U = 0.2$. At the tricritical point P the nature of the phase transition of the easy axis FM (blue region) changes from second to first-order, indicated by dashed and solid lines, respectively. In the red region below the point Q fluctuations stabilize FM order along the hard axis. This region is enclosed by first-order transitions. The inset shows the $T = 0$ phase diagram for the same parameters as a function of inverse interaction strength $1/\rho U$ and crystal field splitting Δ/U .

coupling λ we can approximate $\tilde{m}_2 \approx \frac{J_H}{2U} \tilde{m}_1$. The free energy then becomes a function of \tilde{m}_1 , which to leading order in J_H is the total magnetization of the system. In Fig. 2 the phase diagram is shown as a function of temperature and inverse interaction strength. Due to the non-analytic fluctuation correction $F_{\text{fl}}^{(0)}$, the transition of the easy-plane FM turns first-order at temperatures below the tricritical point P . At the point Q , located at a lower temperature, we find a crossing between the first-order lines calculated from the free energies F_{\perp} and F_{\parallel} for moments in the easy plane and along the hard direction, respectively. Consequently, there is a region below the point Q where fluctuations stabilize magnetic order along the hard axis. As one should expect, the transition at which the moments flip is first-order.

Note that the region of FM order along the hard axis shrinks with increasing band splitting Δ and is completely suppressed beyond a critical value Δ_c (see inset of Fig. 2). Since the competing directional dependent terms $F_{\text{mf}}^{(\lambda)}$ and $F_{\text{fl}}^{(\lambda)}$ are of the same order in the Hund interaction J_H and SO coupling λ , these parameters only have a small effect on the stability of the hard-axis FM.

Depending upon the value of U , which can be viewed as a proxy for tuning parameters like pressure or chemical doping, there are three possible scenarios of phase

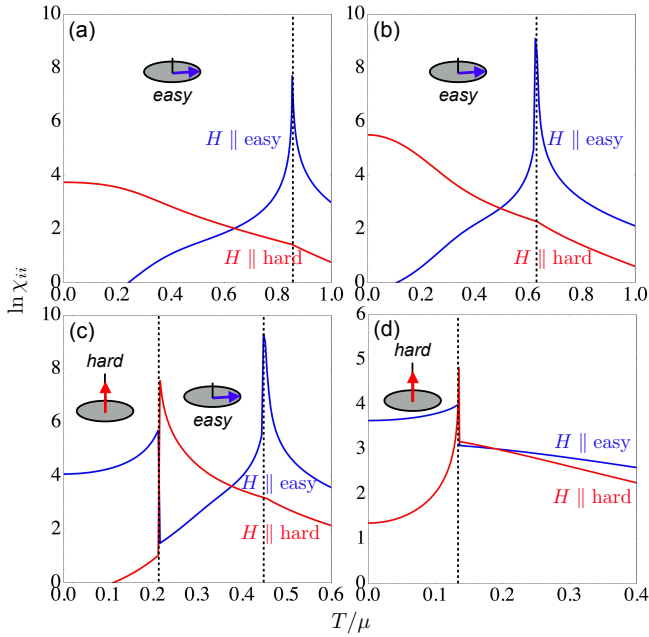


FIG. 3: (color online). Logarithm of the magnetic susceptibilities $\chi_{ii} = \partial m_i / \partial h_i$ for fields in the easy plane (blue) and along the hard axis (red) as a function of temperature for decreasing values of the electron repulsion: $1/(\rho U) = 0.093$ (a), 0.0945 (b), 0.098 (c), 0.103 (d).

transitions as a function of decreasing temperature.

(i) For sufficiently large U , we expect a single transition from a paramagnet to an easy plane FM that is stable down to $T = 0$. Depending on the separation of P and Q this transition could be continuous or first-order.

(ii) Over an intermediate range of U we expect a sequence of two transitions, first from a paramagnet into an easy-plane FM (1st or 2nd order) and then at lower temperatures into a FM state with moments along the hard direction. The transition at which the moment direction switches is always discontinuous.

(iii) Decreasing U further, we find a single first-order transition from a paramagnet into a hard-axis FM.

To make contact with experiments, we calculate the magnetic susceptibilities χ_{\perp} and χ_{\parallel} for fields along the easy and hard directions. In Fig. 3 the evolution of the susceptibilities with temperature is shown for different values of the electron repulsion U . Panels (a) and (b) correspond to case (i) with single continuous transitions into the easy plane FM. At the transition, the easy plane susceptibility χ_{\perp} diverges. The proximity of the system to a fluctuation-driven reorientation of the moments is apparent from the increase in the hard axis susceptibility as the temperature is reduced. Reducing U , this increase becomes more pronounced. Case (ii) is shown in panel (c). At the first-order transition between the two FM states we find an inversion of χ_{\perp} and χ_{\parallel} , characteristic of the switching of the moment direction.

Finally, Fig. 3(d) shows the temperature dependence of the susceptibilities in the regime (iii) where the system exhibits a direct transition from a paramagnet to a FM state with moments along the hard direction, exactly as it has been observed experimentally [11, 12]. We find a similar characteristic crossing of susceptibilities at a temperature T^* slightly above the ordering temperature T_c . As we increase U and move closer towards the point Q (see Fig. 2), the temperatures T^* and T_c merge.

Discussion and Conclusions. We have described a generic mechanism whereby transverse quantum fluctuations drive unexpected magnetic behavior in itinerant FMs with magnetic anisotropy. At low temperatures, these systems can magnetize along directions that are unfavorable at higher temperatures. As a proof of principle we have explicitly demonstrated this for a simple two-band model with an easy-plane anisotropy generated by SO coupling. We found that close to the quantum critical point the moments switch to the magnetic hard axis in order to maximize the phase space for transverse spin fluctuations – a ‘quantum indian rope trick’! [30]. While we treat the SO perturbatively, the enhancement of hard-axis ordering with increasing SO coupling suggests that our results also apply to systems with strong SO interaction.

Related phase transitions are also possible. Quantum fluctuations could also stabilize a modulated spiral state which preempts the first-order transition into the homogeneous FM [17, 19]. Since planar spirals are compatible with magnetic easy-plane anisotropies, one might find systems that show spiral order which then becomes unstable toward a hard-axis FM at even lower temperatures. For a tetragonal system with magnetic easy-axis anisotropy we expect a similar moment reorientation as for the easy-plane case studied in this Letter. Simply because by flipping the moments into the hard plane, the easy direction becomes available for transverse spin fluctuations. Orthorhombic systems with three inequivalent magnetic directions might offer even more interesting phase behavior with a two-step moment reorientation.

With recent experimental advances it is now possible to study the phase reconstruction near quantum critical points down to extremely low temperatures. There are several materials in which FM ordering along a hard magnetic direction has been discovered. The most promising is YbNi_4P_2 [12] because of its close proximity to a quantum phase transition. The observed crossing of susceptibilities slightly above the ordering temperature is in qualitative agreement with our prediction based on a simple itinerant model. However, the measured critical exponents are inconsistent with a pure itinerant model [12]. In fact, YbNi_4P_2 exhibits quasi one-dimensional spin chains, which couple to the conduction electrons, and is a Kondo-lattice system with strong interactions between conduction- and localized f -electrons. While the non-analytic free energy corrections are expected to

have similar effects in systems with local moments [16], a quantitative analysis based on a realistic microscopic model for YbNi_4P_2 would be desirable.

We point out that the suggested mechanism for the moment switching close to FM quantum critical points does not require frustration. It is expected to be generic since it stems from the same non-analytic free energy corrections that are responsible for fluctuation-induced first-order behavior in practically all itinerant FMs, irrespective of microscopic details. It is possible to go beyond our simple model to include frustration, the coupling to local moments, and potentially even Kondo physics. The interplay of these extra ingredients with electronic low energy-fluctuations might yield a plethora of novel and exciting ordering phenomena.

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- $$\Gamma_{\nu_1 \nu_2 \nu_3}^{(1)} = \frac{D_{\nu_1 \nu_2}^{(1)} \Pi_{\nu_1 \nu_2}^{(1)}}{2\zeta_{\nu_2 \nu_3}} - \frac{(\Pi_{\nu_1 \nu_2}^{(1)} - \tilde{\Pi}_{\nu_1 \nu_2})}{\zeta_{\nu_2 \nu_3}^2}$$
- $$\zeta_{\nu_2 \nu_3} = \Delta + U(\nu_2 \tilde{m}_1 - \nu_3 \tilde{m}_2),$$
- where the derivative operators $D_{\nu\nu'}^{(1)} = \partial_{\mu_1}$ and $D_{\nu\nu'}^{(1)} = \partial_{\mu_1} - \frac{\nu}{U} \partial_{\tilde{m}_1}$ and $\tilde{\Pi}_{\nu_1 \nu_2}(q) = T \sum_{\omega} \int_{\mathbf{k}} G_{1, \nu_1}(k + q) G_{2, \nu_2}(k)$. Substituting into the fluctuation term gives
- $$F_{\text{fl}}^{(\lambda)} \approx -\frac{\lambda^2 U^2 T}{4\Delta^2} (\mathcal{P}_{\parallel} - \mathcal{P}_{\perp}) \tilde{m}_2 \partial_{\tilde{m}_1} \sum_{\omega} \int_{\mathbf{q}} \Pi_{+-}^{(1)} \Pi_{-+}^{(1)}$$
- which shows that the dominant terms originate from transverse spin fluctuations and leads to the result given in the text.
- [30] In the extreme version of the Indian-RopeTrick a rope rises from a basket and a helper of the Fakir then climbs up the rope. Finally, the helper would descend or even disappear from view. This is either a hoax or true magic. It is however possible to perform an Indian-Rope Trick with a chain of inverted pendula, stabilized by vibrations of the base point [31, 32].
- [31] D. J. Acheson, *Proc. R. Soc. A* **443**, 239 (1993).
- [32] D. J. Acheson and T. Mullin, *Nature* **336**, 215 (1993).